

Title: TCS1 Transfer Functions**Author: Eric Warmbier****Description:****This document derives the various transfer functions for the TCS1 system on the IRTF.****The system is broken down into blocks in a Visio document. A transfer function will be developed for these blocks (or groups of blocks) to be used in Simulink. Not all blocks will be present in this document. Simple blocks may only be presented in the Visio file.****Constants:**

$$\mu := 1 \cdot 10^{-6} \quad n := 1 \cdot 10^{-9} \quad p := 1 \cdot 10^{-12}$$

$$k := 1 \cdot 10^3 \quad as := \frac{\pi}{648000} \text{rad}$$

BLOCK #1 & #2**These blocks are inverting summing amplifiers with some filtering and some compensation on one input. Each input can be calculated separately and then summed.****Generically, the transfer function is:**

$$G1_comp(s) = \frac{-Z_feedback(s)}{Z_source(s)}$$

Where:

$$Z_G1_comp_feedback(s) := \frac{300k \cdot \left(\frac{1}{s \cdot 2200p} \right)}{300k + \left(\frac{1}{s \cdot 2200p} \right)}$$

$$Z_G1_comp_source(s) := \left[\frac{1}{s \cdot 0.33\mu} + 56k + \frac{68k \cdot \left(\frac{1}{s \cdot 0.068\mu} \right)}{68k + \left(\frac{1}{s \cdot 0.068\mu} \right)} \right]$$

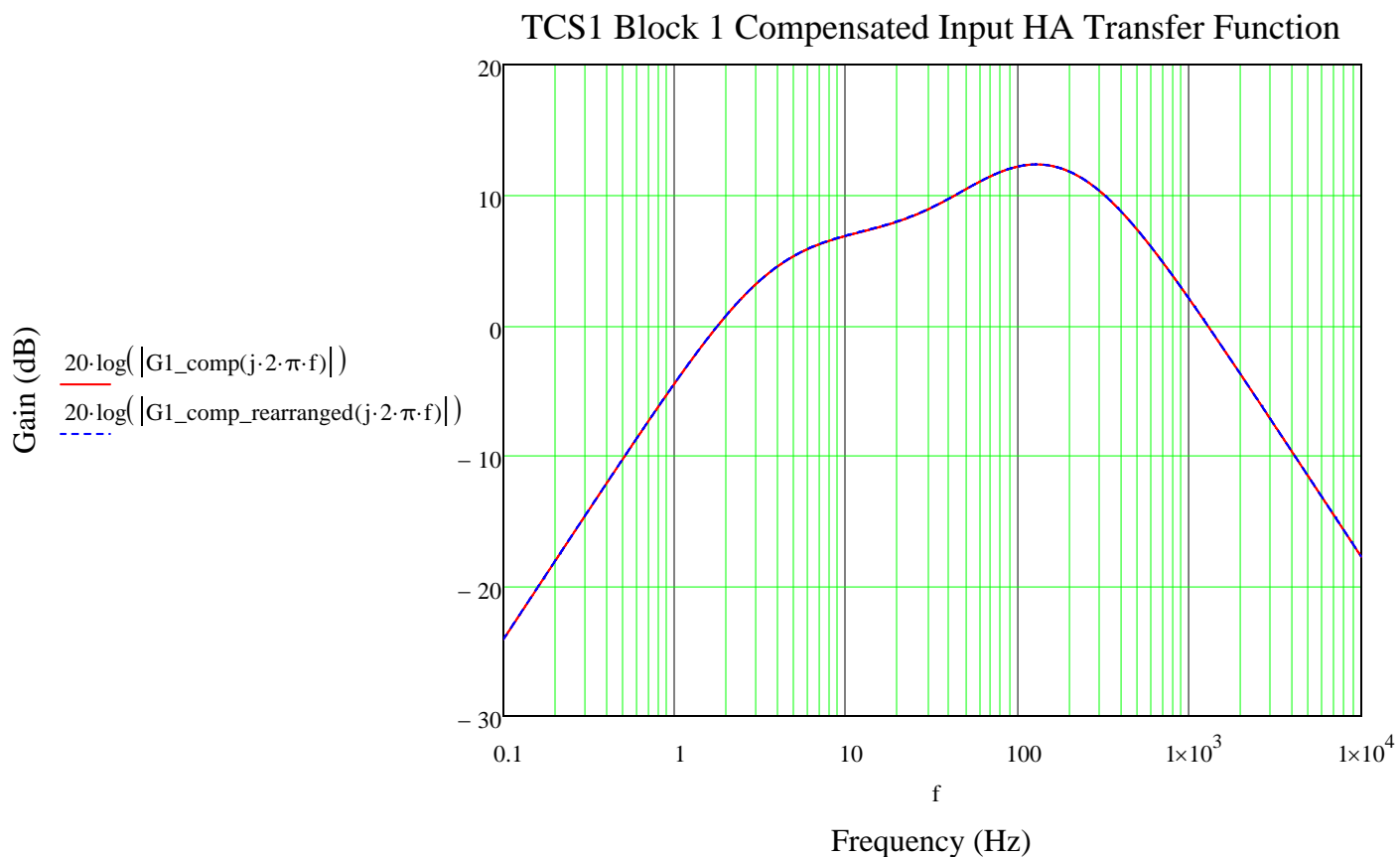
$$G1_comp(s) := \frac{-Z_G1_comp_feedback(s)}{Z_G1_comp_source(s)}$$

$$G1_comp(s) \left| \begin{array}{l} \text{float, 3} \\ \text{collect, s} \end{array} \right. \rightarrow - \frac{5.26e26 \cdot s + 2.43e24 \cdot s^2}{2.45e26 \cdot s + 6.13e23 \cdot s^2 + 2.99e20 \cdot s^3 + 5.31e27}$$

Rearrange the function into a more convenient form and verify that it is still equal to the original.

$$G1_comp_rearranged(s) := \frac{457.63 \cdot 10^{-6} s^2 + 0.0991s}{56.309 \times 10^{-9} \cdot s^3 + 115.44 \times 10^{-6} \cdot s^2 + 0.0461 \cdot s + 1}$$

Graph to verify that the function is similar to what is expected. It is a bandpass filter with some compensation.



Generically, the other input transfer function is:

$$G1_other(s) = \frac{-Z_feedback(s)}{Z_source(s)}$$

Where:

$$Z_G1_other_feedback(s) := \frac{300k \cdot \left(\frac{1}{s \cdot 2200p} \right)}{300k + \left(\frac{1}{s \cdot 2200p} \right)}$$

$$Z_{G1_other_source}(s) := 100k$$

$$G1_other(s) := \frac{-Z_{G1_other_feedback}(s)}{Z_{G1_other_source}(s)}$$

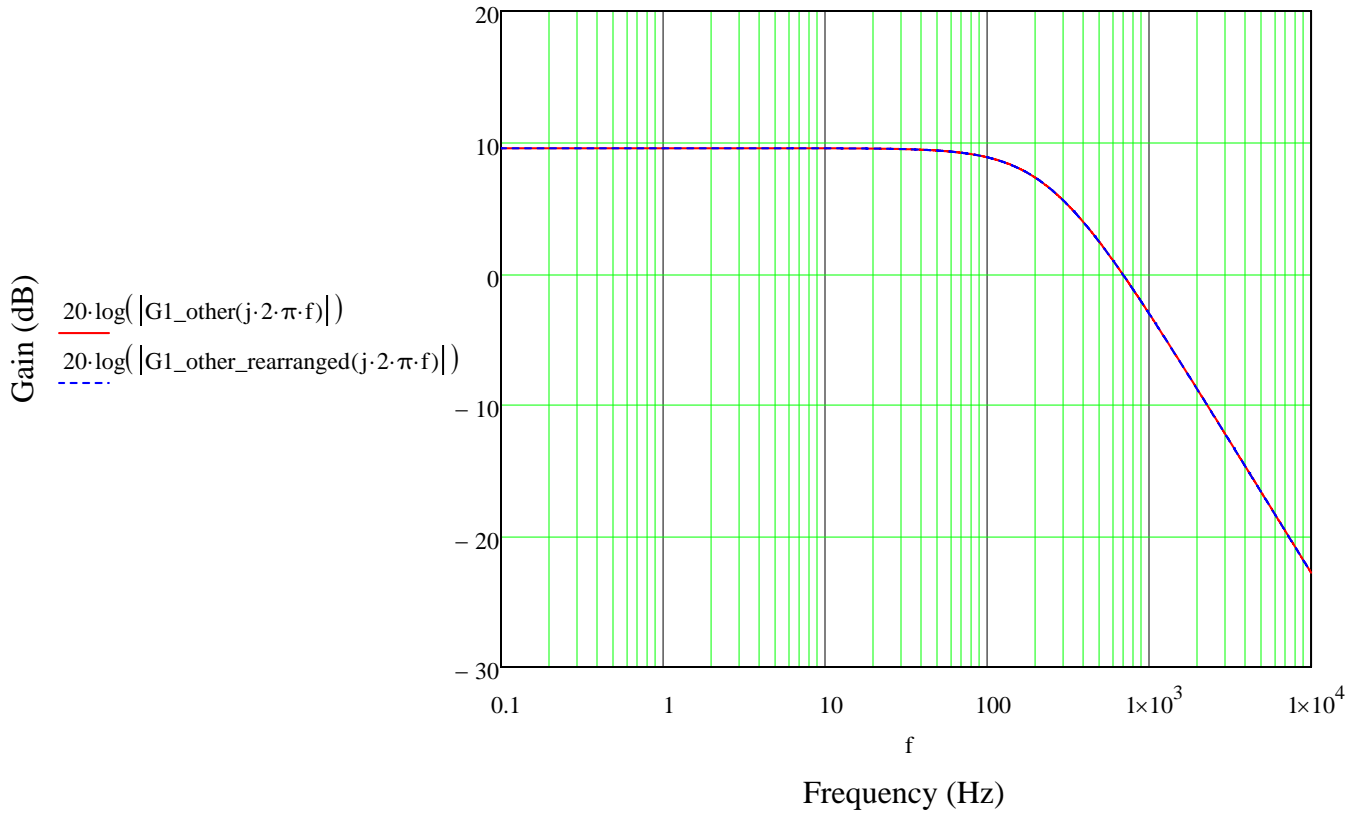
$$G1_other(s) \left| \begin{array}{l} \text{collect, s} \\ \text{float, 3} \end{array} \right. \rightarrow -\frac{150000.0}{33.0 \cdot s + 50000.0}$$

Rearrange the function into a more convenient form and verify that it is still equal to the original.

$$G1_other_rearranged(s) := -\frac{3}{660 \cdot 10^{-6} \cdot s + 1}$$

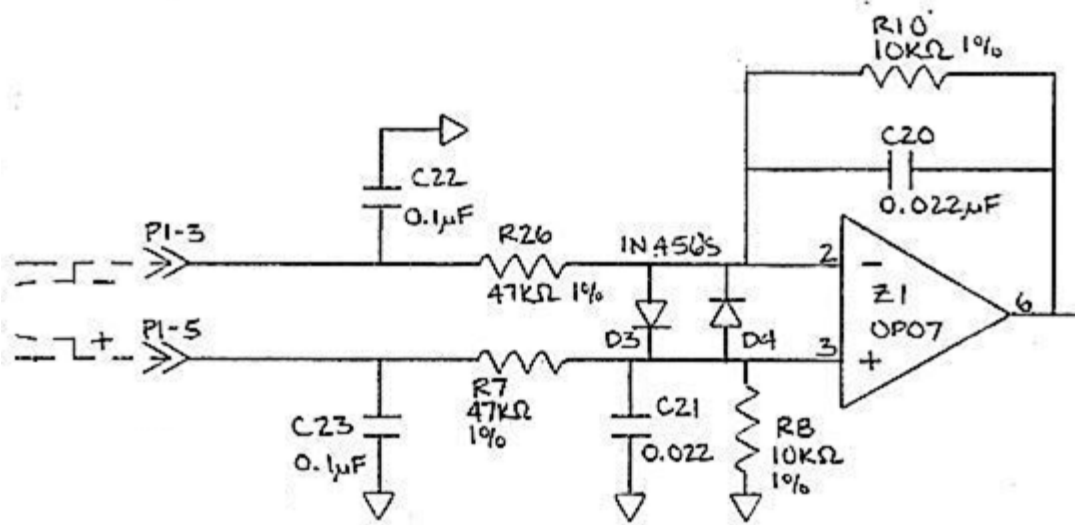
Graph to verify that the function is similar to what is expected. It is a lowpass filter.

TCS1 Block 1 Other Input HA Transfer Function



BLOCK #3

These blocks are difference amplifiers with some low pass filtering.



Generically, the transfer function is:

$$G3(s) = \frac{Z_{\text{feedback}}(s)}{Z_{\text{source}}(s)}$$

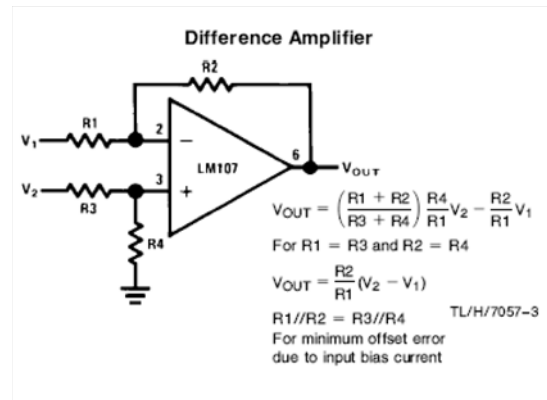
Where:

$$Z_{\text{G3_feedback}}(s) := \frac{10k \cdot \left(\frac{1}{s \cdot 0.022\mu} \right)}{10k + \left(\frac{1}{s \cdot 0.022\mu} \right)}$$

$$Z_{\text{G3_source}}(s) := 47k$$

$$G3(s) := \frac{Z_{\text{G3_feedback}}(s)}{Z_{\text{G3_source}}(s)}$$

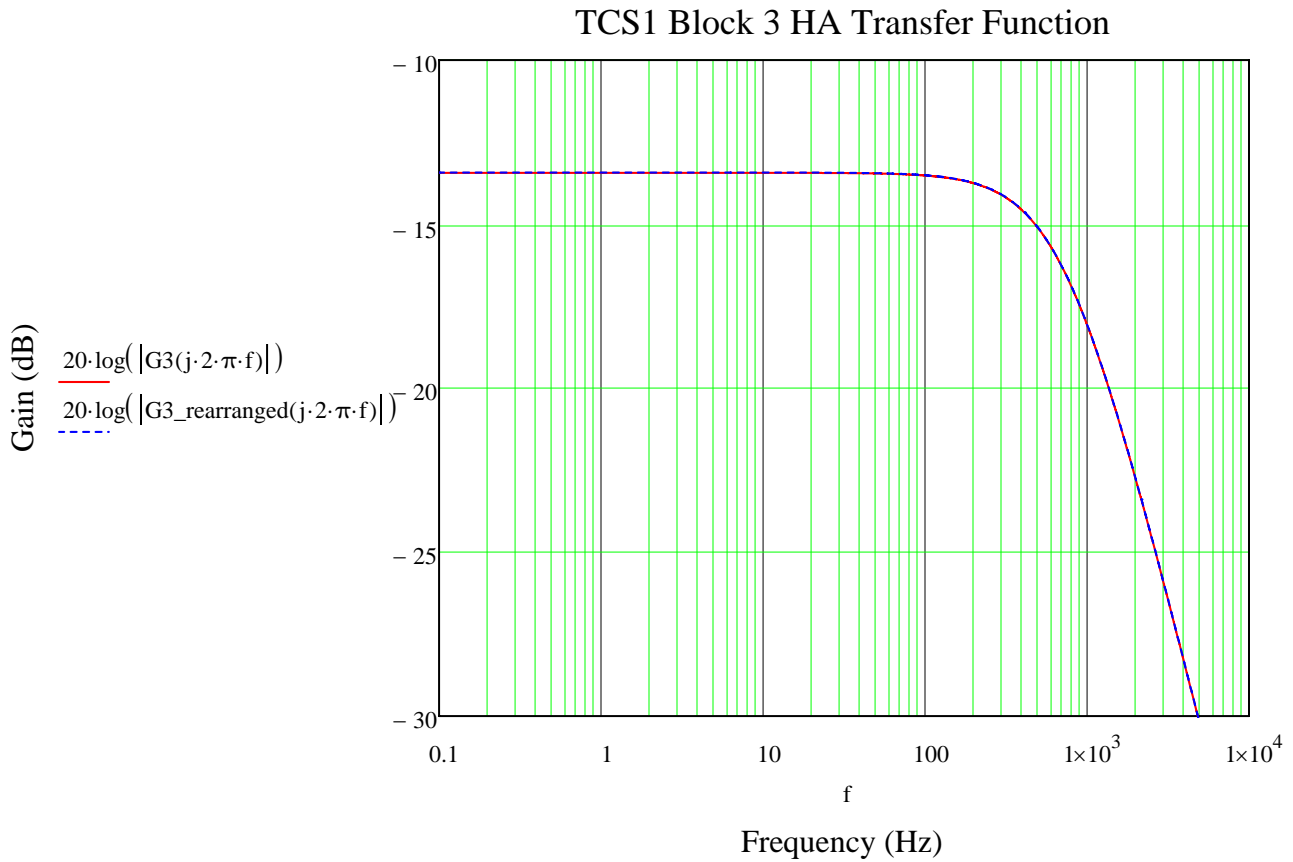
$$G3(s) \left| \begin{array}{l} \text{collect} \\ \text{float, 3} \end{array} \right. \rightarrow \frac{9671.0}{10.0 \cdot s + 45450.0}$$



Rearrange the function into a more convenient form and verify that it is still equal to the original.

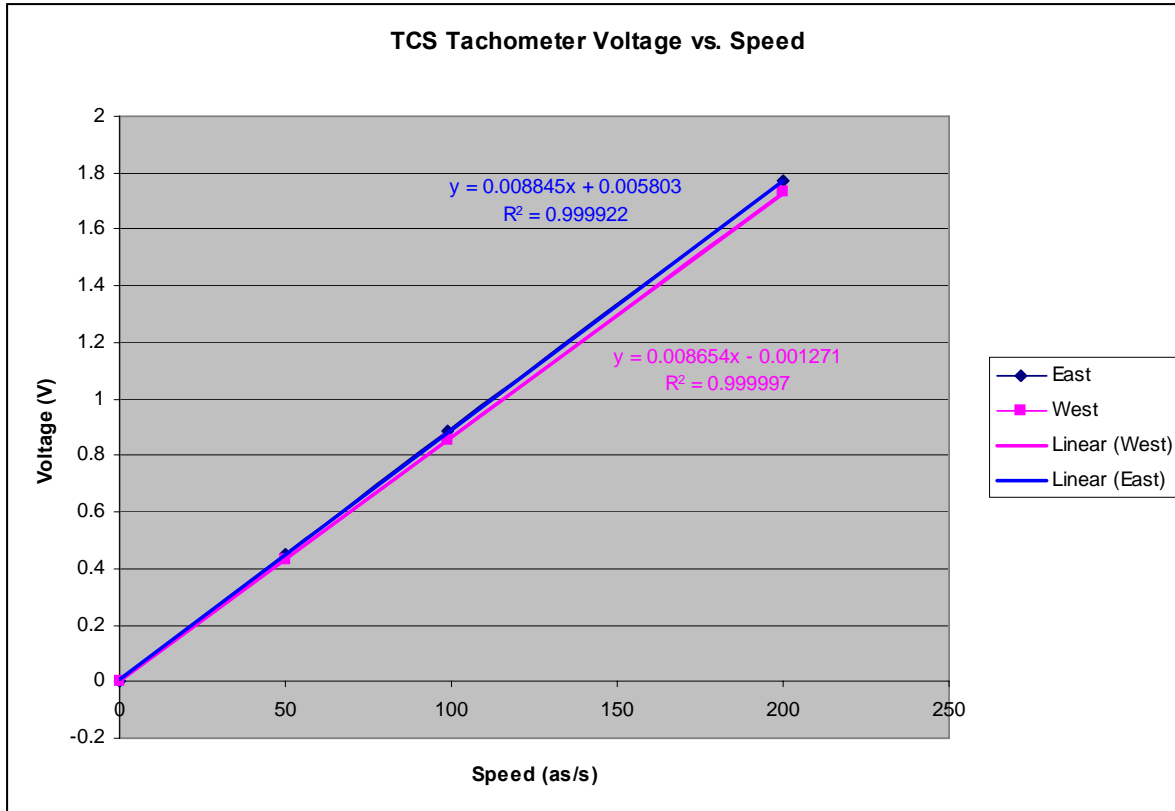
$$G3_rearranged(s) := \frac{0.213}{220 \cdot 10^{-6} \cdot s + 1}$$

Graph to verify that the function is similar to what is expected. It is a low pass with less than unity gain.



The tachometers also have a gain (conversion from motion, in radians, to volts).

From testing, the results below were obtained.



Assume that a "nominal" tach will have a gain equal to the average of these two slopes.

$$\text{nom_tach_gain} := \frac{\left[0.008845 \frac{\text{V}}{\left(\frac{\text{as}}{\text{s}} \right)} + 0.008654 \frac{\text{V}}{\left(\frac{\text{as}}{\text{s}} \right)} \right]}{2}$$

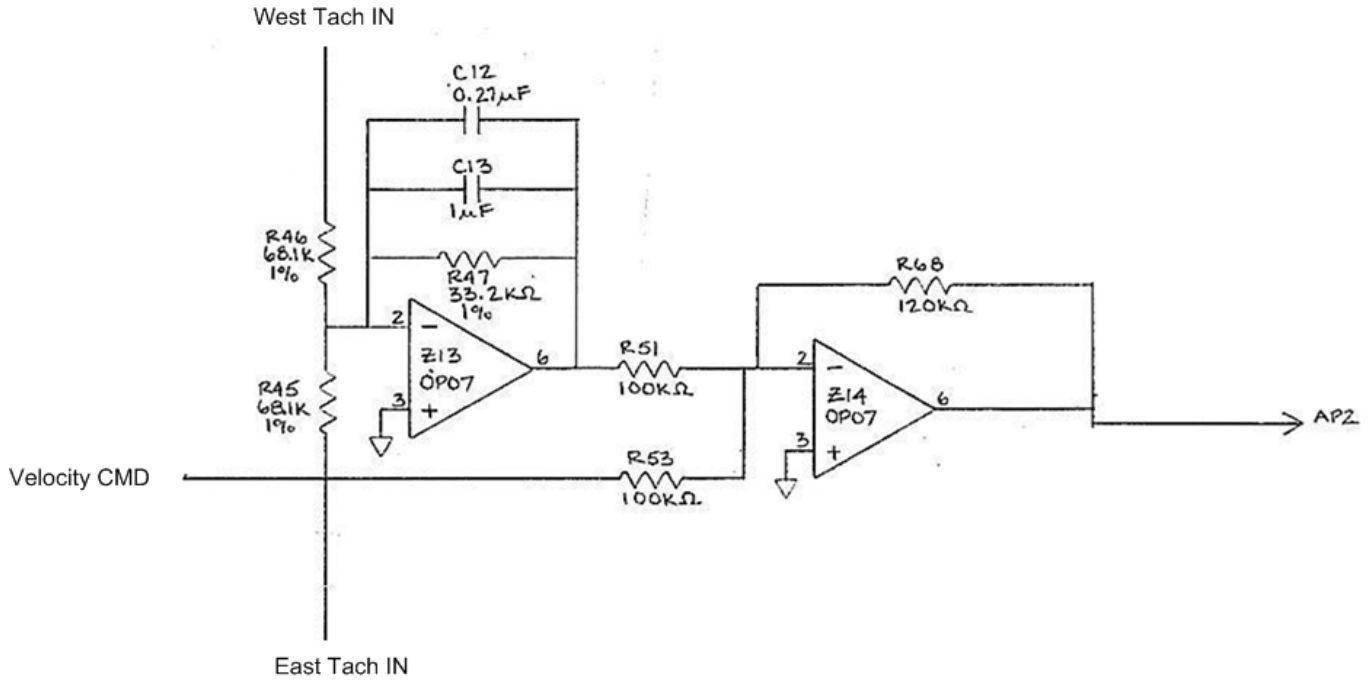
$$\text{nom_tach_gain} = 0.0087 \cdot \frac{\text{V}}{\left(\frac{\text{as}}{\text{s}} \right)}$$

The units above are in V(arcsecond/s). The input will be in radians/s, so this need to be converted into V/(radians/s).

$$\text{nom_tach_gain} = 1804.7139 \cdot \frac{\text{V}}{\left(\frac{\text{rad}}{\text{s}} \right)}$$

BLOCK #4

These blocks are inverting summing amplifiers with some filtering on the first stage. A transfer function for each input can be determined and then added together.



Generically, the transfer function for each of the first stage inputs (Z13) is:

$$G4(s) = \frac{-Z_{\text{feedback}}(s)}{Z_{\text{source}}(s)}$$

Where:

$$Z_{G4_feedback}(s) := \frac{33.2k \cdot \left(\frac{1}{s \cdot 1.47\mu} \right)}{33.2k + \left(\frac{1}{s \cdot 1.47\mu} \right)}$$

$$Z_{G4_source}(s) := 68.1k$$

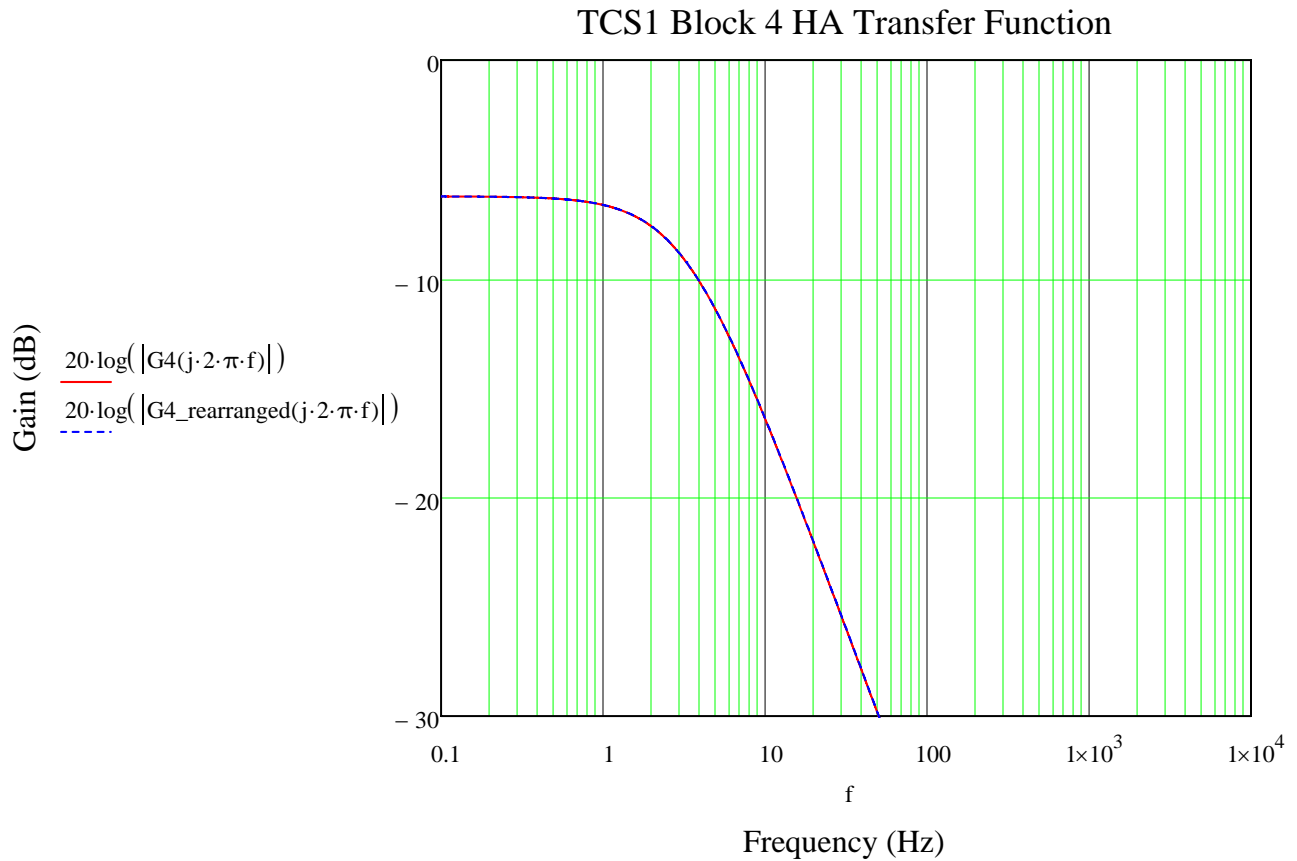
$$G4(s) := \frac{-Z_{G4_feedback}(s)}{Z_{G4_source}(s)}$$

$$G4(s) \left| \begin{array}{l} \text{float, 3} \\ \text{collect, s} \end{array} \right. \rightarrow -\frac{1.36e9}{1.36e8 \cdot s + 2.79e9}$$

Rearrange the function into a more convenient form and verify that it is still equal to the original.

$$G4_rearranged(s) := -\frac{0.4875}{0.0487s + 1}$$

Graph to verify that the function is similar to what is expected - a low pass filter with gain of ~-0.5.



BLOCK #7

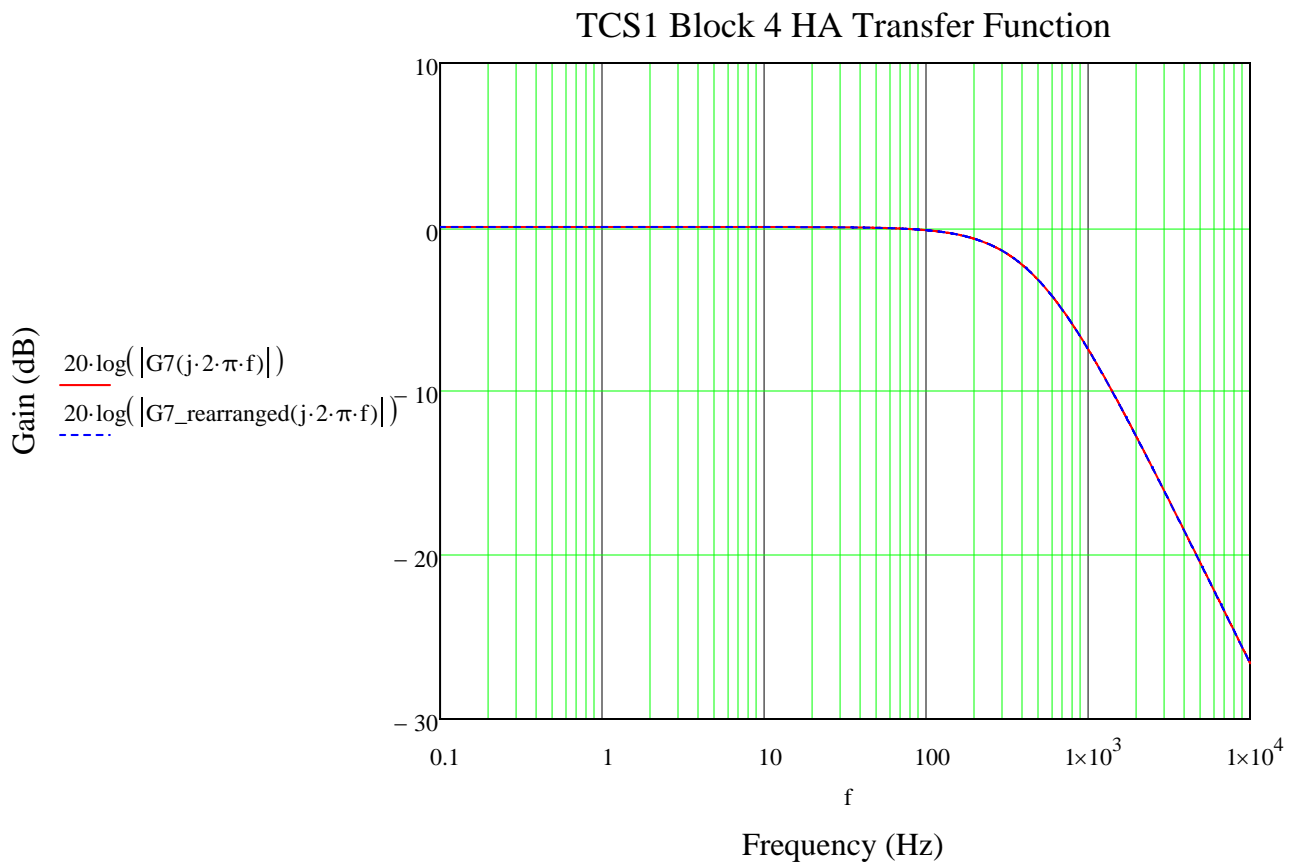
The block is a group of inverting summing amplifiers. There are two stages. The first stage has two inverting amplifiers without filtering that are summed into the final stage that has filtering. The only stage that will be calculated is the final stage for only the command input. The mathematics behind this circuit creates a magnitude output limited circuit with a gain of 1. The first stages are just simple addition.

$$G7(s) \left| \begin{array}{l} \text{float, 3} \\ \text{collect, s} \end{array} \right. \rightarrow -\frac{2.94e8}{100000.0 \cdot s + 2.94e8}$$

Rearrange the function into a more convenient form and verify that it is still equal to the original.

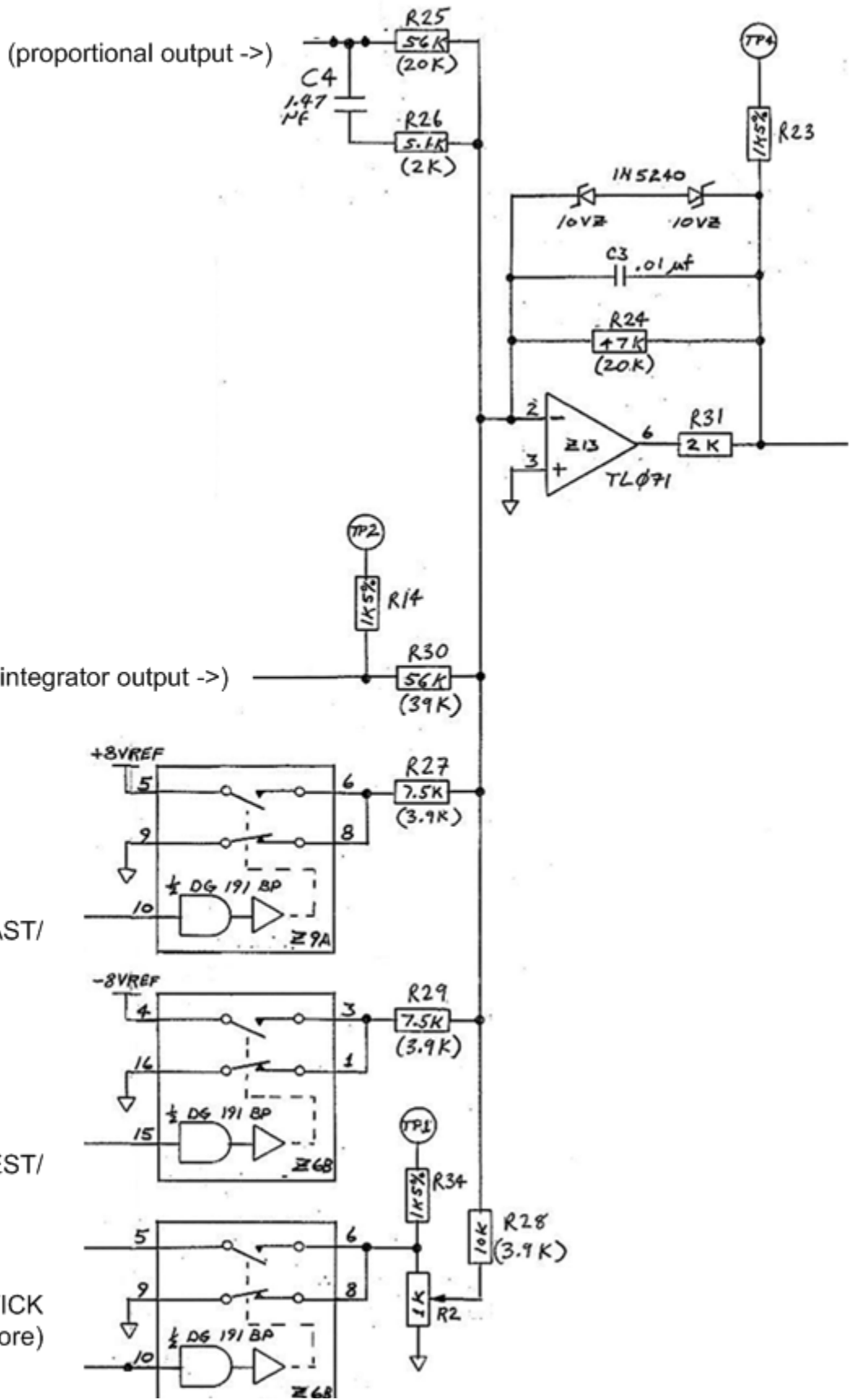
$$G7_rearranged(s) := -\frac{1}{340 \cdot 10^{-6} s + 1}$$

Graph to verify that the function is similar to what is expected - a low pass filter with gain of 1 (0dB).



BLOCK #9

The block is an inverting summing amplifier with 5 inputs, some filtering, and output magnitude limiting. The "Joystick" input will be ignored since it will not be used in the servo analysis.





Generically, the transfer function for each input is:

$$G9(s) = \frac{-Z_{\text{feedback}}(s)}{Z_{\text{source}}(s)}$$

Where:

$$Z_{G9_feedback}(s) := \frac{47k \cdot \left(\frac{1}{s \cdot 0.01\mu} \right)}{47k + \left(\frac{1}{s \cdot 0.01\mu} \right)}$$

The source impedance for each input will vary for each input.

For "proportional output":

$$Z_{G9_prop_source}(s) := \frac{56k \cdot \left(\frac{1}{s \cdot 1.47\mu} + 5.1k \right)}{56k + \left(\frac{1}{s \cdot 1.47\mu} + 5.1k \right)}$$

$$G9_{\text{proportional}}(s) := \frac{-Z_{G9_feedback}(s)}{Z_{G9_prop_source}(s)}$$

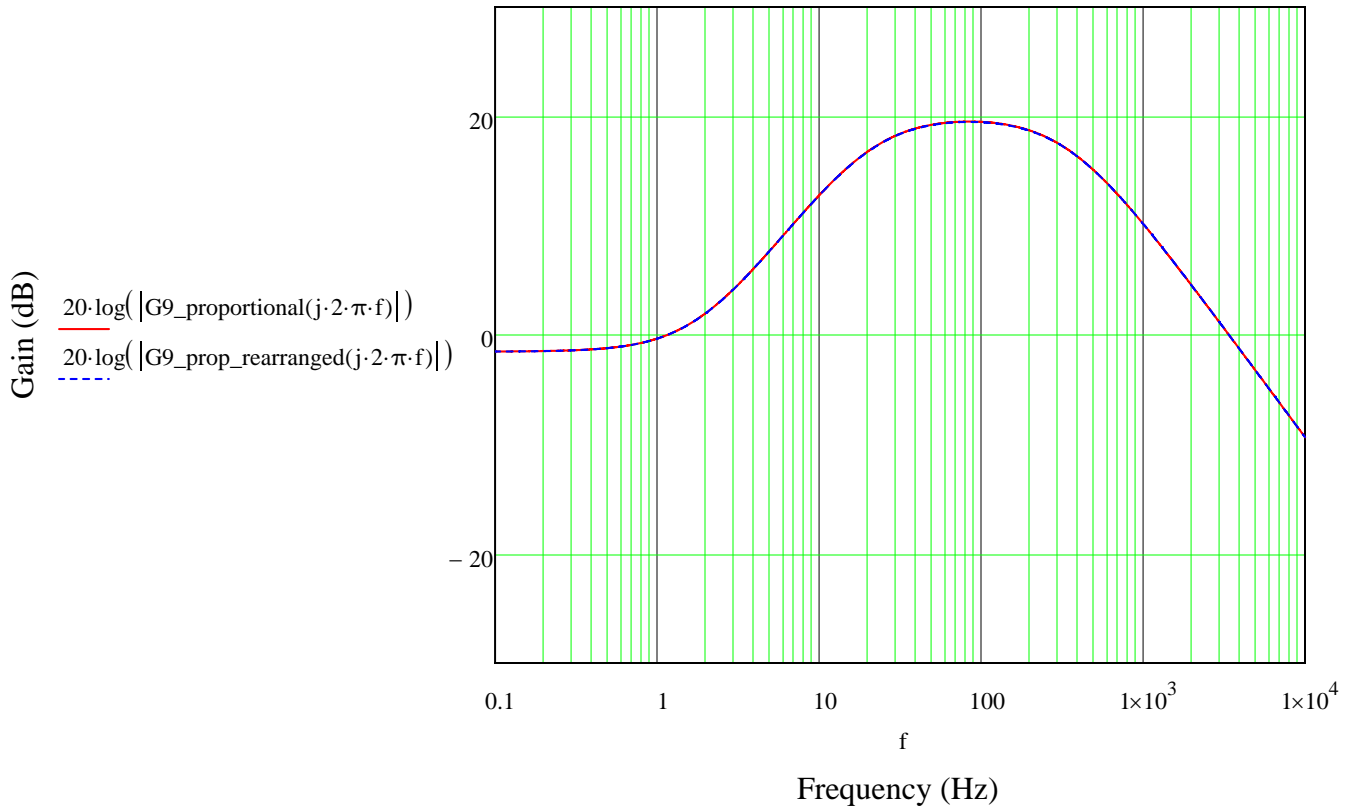
$$G9_{\text{proportional}}(s) \left| \begin{array}{l} \text{float, 3} \\ \text{collect, s} \end{array} \right. \rightarrow -\frac{5.61e14 \cdot s + 6.24e15}{5.93e13 \cdot s + 2.62e10 \cdot s^2 + 7.44e15}$$

Rearrange the function into a more convenient form and verify that it is still equal to the original.

$$G9_{\text{prop_rearranged}}(s) := -\frac{0.0754s + 0.8387}{3.522 \cdot 10^{-6} s^2 + 0.008s + 1}$$

Graph to verify that the function is similar to what is expected - a low pass filter with another pole.

TCS1 Block 4 HA Transfer Function



For "integrator output":

$$Z_{G9_int_source}(s) := 56k$$

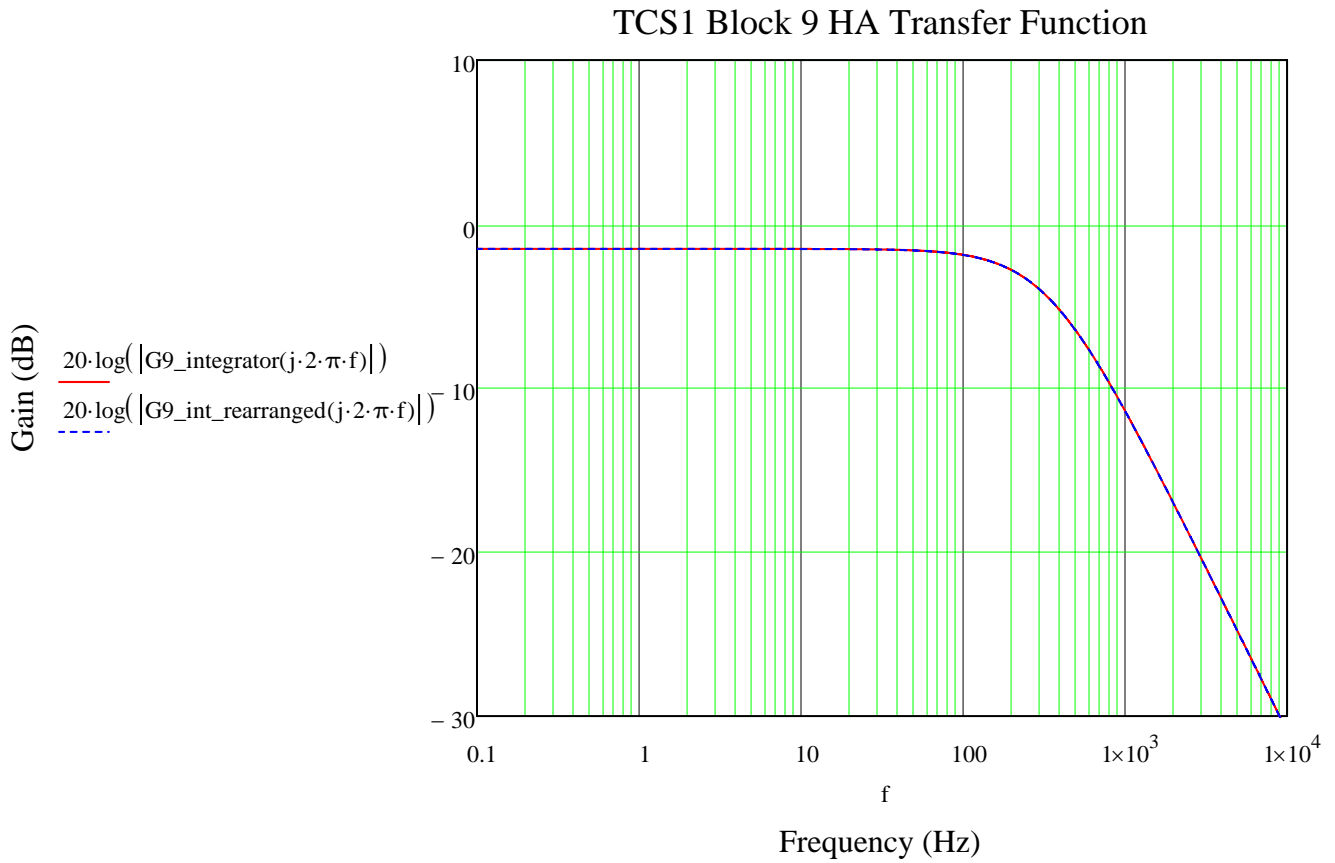
$$G9_integrator(s) := \frac{-Z_{G9_feedback}(s)}{Z_{G9_int_source}(s)}$$

$$G9_integrator(s) \left| \begin{array}{l} \text{float, 3} \\ \text{collect, s} \end{array} \right. \rightarrow -\frac{5.37e8}{300800.0 \cdot s + 6.4e8}$$

Rearrange the function into a more convenient form and verify that it is still equal to the original.

$$G9_int_rearranged(s) := -\frac{0.8391}{470 \cdot 10^{-6} s + 1}$$

Graph to verify that the function is similar to what is expected - a low pass filter.



For "EAST and WEST" slew:

$$Z_G9_slew_source(s) := 7.5k$$

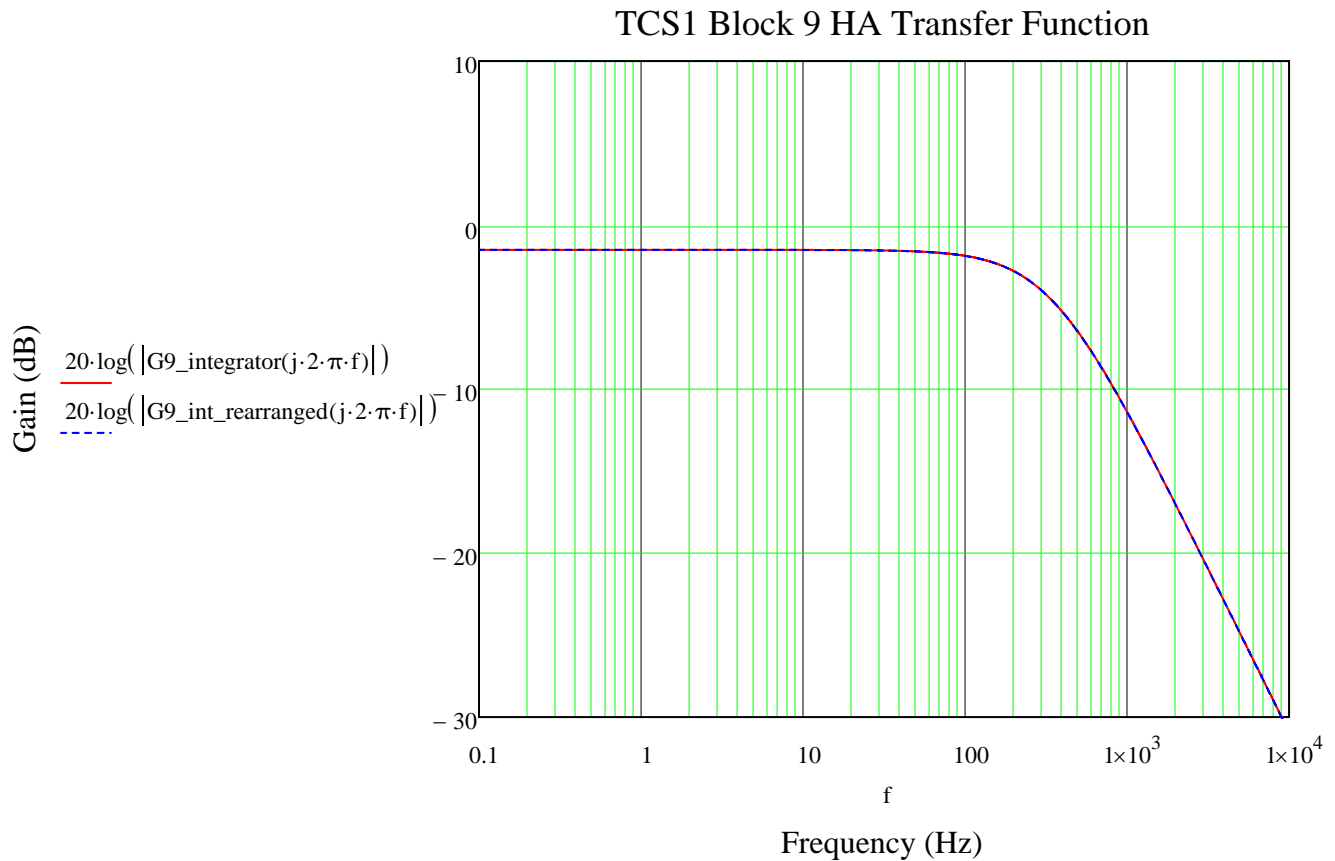
$$G9_slew(s) := \frac{-Z_G9_feedback(s)}{Z_G9_slew_source(s)}$$

$$G9_slew(s) \left| \begin{array}{l} \text{float, 3} \\ \text{collect, s} \end{array} \right. \rightarrow -\frac{1.25e9}{94000.0 \cdot s + 2.0e8}$$

Rearrange the function into a more convenient form and verify that it is still equal to the original.

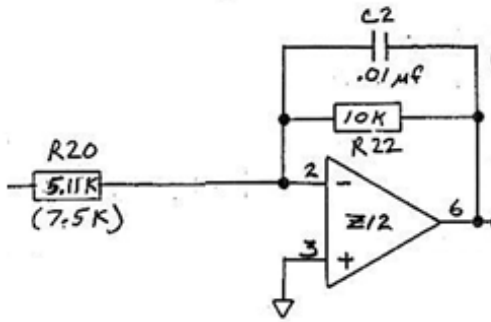
$$G9_slew_rearranged(s) := \frac{6.25}{470 \cdot 10^{-6} s + 1}$$

Graph to verify that the function is similar to what is expected - a low pass filter.



BLOCK #11

This block is an inverting amplifier with filtering:



Generically, the HA axis transfer function is:

$$G_{11}(s) = \frac{-Z_{\text{feedback}}(s)}{Z_{\text{source}}(s)}$$

Where:

$$Z_{G11_HA_feedback}(s) := \frac{10k \cdot \left(\frac{1}{s \cdot 0.01\mu} \right)}{10k + \left(\frac{1}{s \cdot 0.01\mu} \right)}$$

$$Z_{G11_HA_source}(s) := 5.11k$$

$$G_{11_HA}(s) := \frac{-Z_{G11_HA_feedback}(s)}{Z_{G11_HA_source}(s)}$$

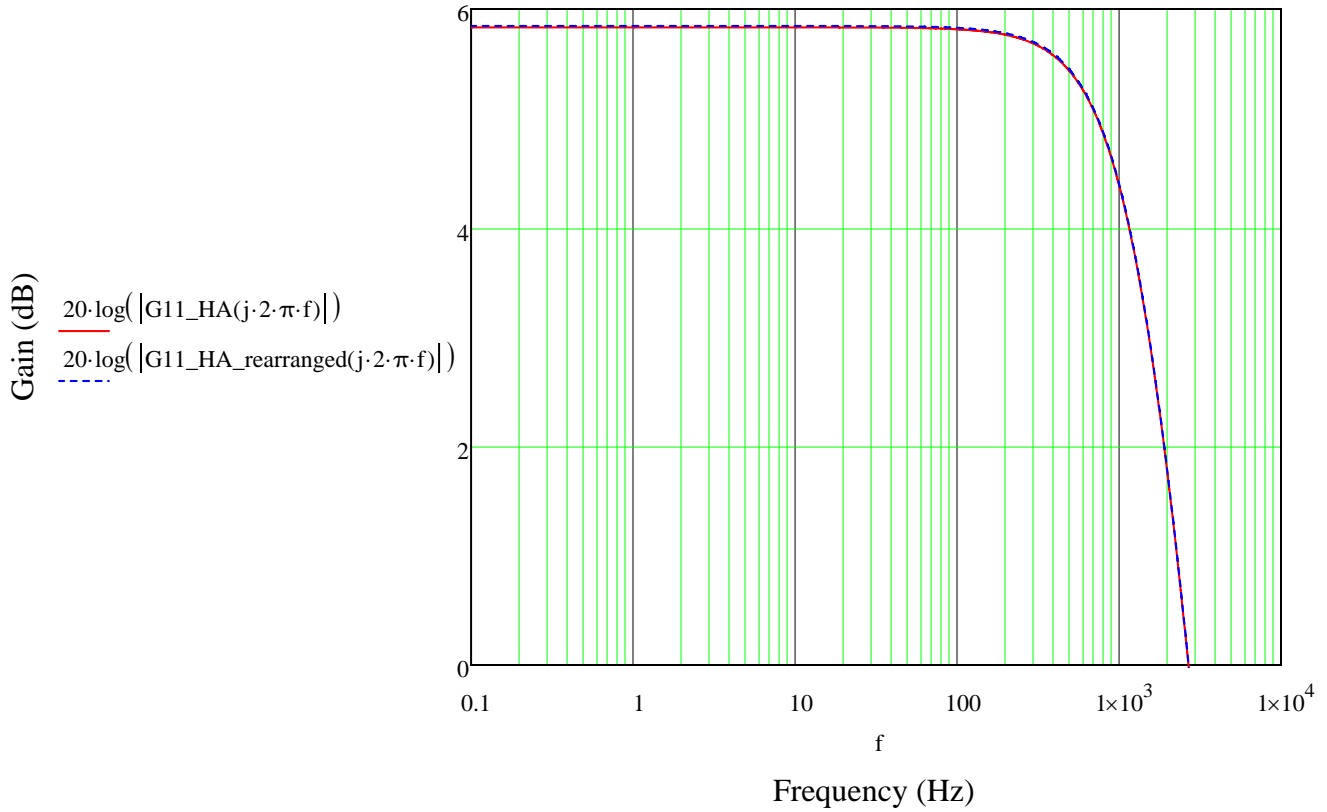
$$G_{11_HA}(s) \left| \begin{array}{l} \text{float, 3} \\ \text{collect, s} \end{array} \right. \rightarrow -\frac{3.13e9}{160000.0 \cdot s + 1.6e9}$$

Rearrange the function into a more convenient form and verify that it is still equal to the original.

$$G_{11_HA_rearranged}(s) := -\frac{1.96}{100 \cdot 10^{-6} s + 1}$$

Graph to verify that the function is similar to what is expected - a low pass filter with gain.

TCS1 Block 11 HA Transfer Function



The DEC axis has a slightly different gain. Generically, the transfer function is:

$$G(s) = \frac{-Z_{\text{feedback}}(s)}{Z_{\text{source}}(s)}$$

Where:

$$Z_{G11_DEC_feedback}(s) := \frac{10k \cdot \left(\frac{1}{s \cdot 0.01\mu} \right)}{10k + \left(\frac{1}{s \cdot 0.01\mu} \right)}$$

$$Z_{G11_DEC_source}(s) := 7.5k$$

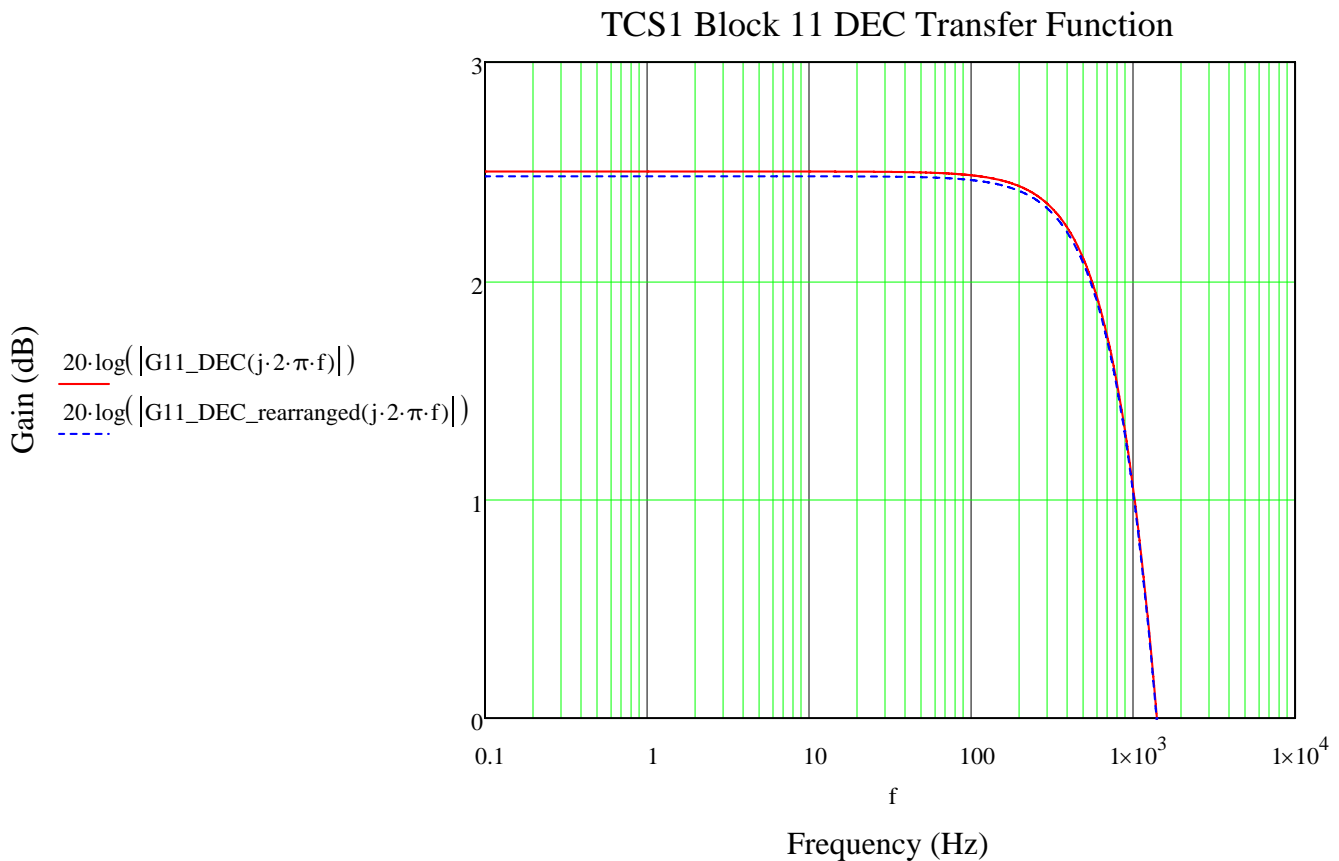
$$G_{11_DEC}(s) := \frac{-Z_{G11_DEC_feedback}(s)}{Z_{G11_DEC_source}(s)}$$

$$G11_DEC(s) \left| \begin{array}{l} \text{float, 3} \\ \text{collect, s} \end{array} \right. \rightarrow -\frac{2.13e9}{160000.0 \cdot s + 1.6e9}$$

Rearrange the function into a more convenient form and verify that it is still equal to the original.

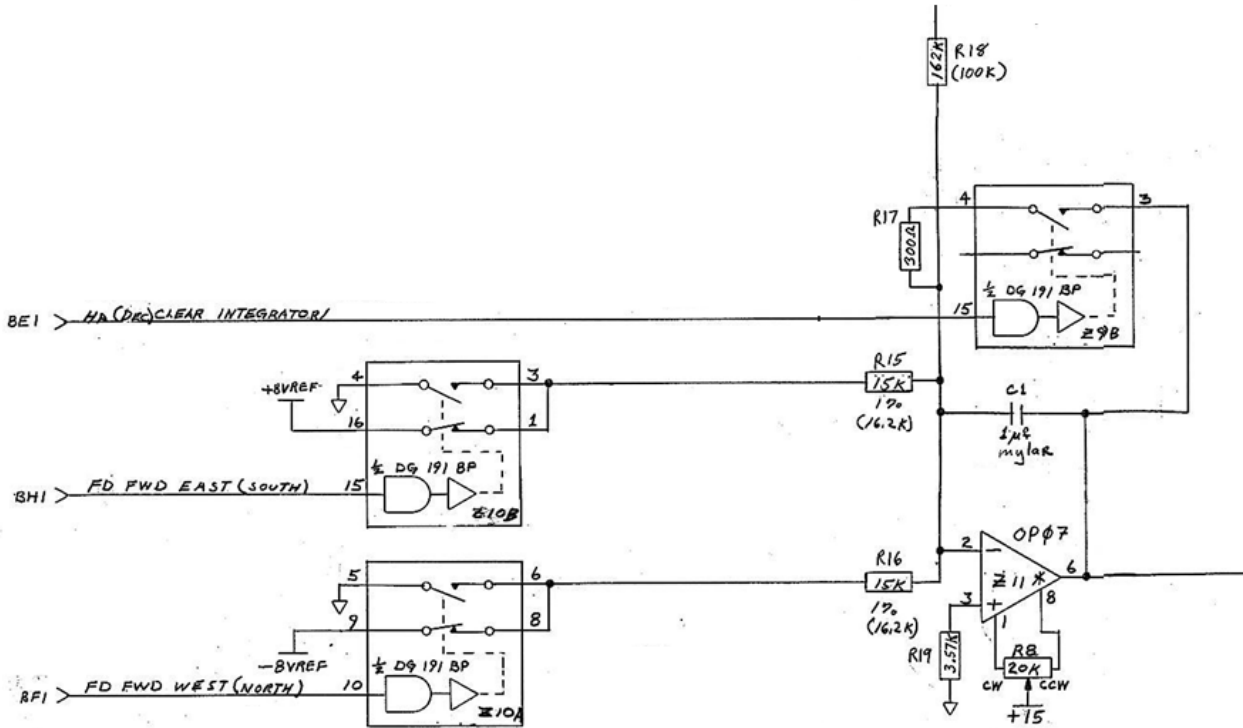
$$G11_DEC_rearranged(s) := \frac{1.33}{0.0001 \cdot s + 1.0}$$

Graph to verify that the function is similar to what is expected - a low pass filter with gain.



BLOCK #12

This block consists of a summing, inverting integrator that can be cleared (cap shorted with 300Ω). Each input will be calculated separately and these results can be summed together for the complete function. When the integrator is put in parallel with a resistor, the gains for all the inputs become very small and the filter formed has a relatively low cutoff frequency. Normal operation is with the capacitor NOT in parallel with R17.



ALL calculations below are with no R17 in parallel with the capacitor (normal operation) until noted.

Generically, the transfer function for each input is:

$$G(s) = \frac{-Z_{\text{feedback}}(s)}{Z_{\text{source}}(s)}$$

Where:

$$Z_{12_feedback}(s) := \frac{1}{s \cdot 1\mu}$$

For the error input:

$$Z_{12_error_source}(s) := 162k$$

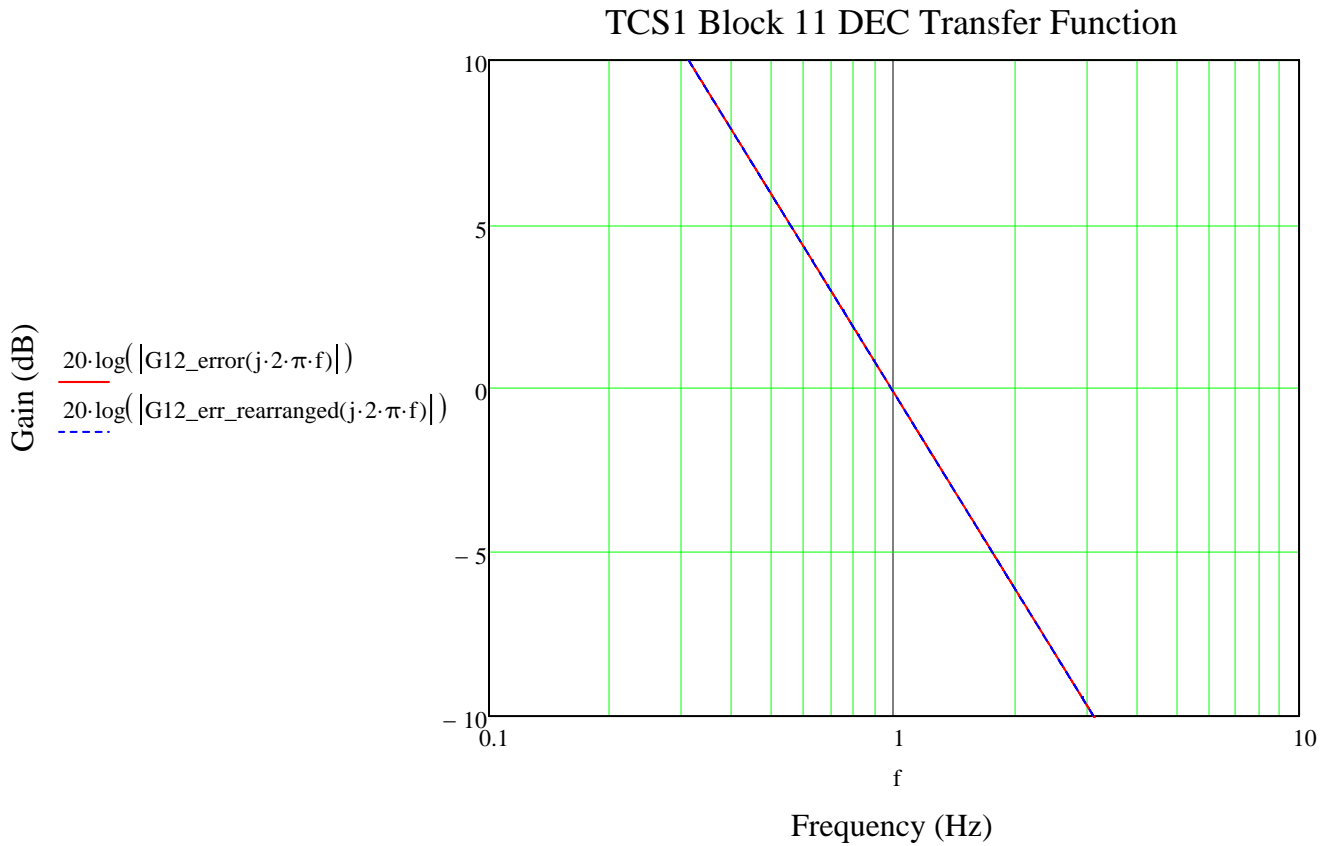
$$G_{12_error}(s) := \frac{-Z_{12_feedback}(s)}{Z_{12_error_source}(s)}$$

$$G12_error(s) \left| \begin{array}{l} \text{float, 4} \\ \text{collect, s} \end{array} \right. \rightarrow -\frac{6.173}{s}$$

Rearrange the function into a more convenient form and verify that it is still equal to the original.

$$G12_err_rearranged(s) := -\frac{1}{0.162s}$$

Graph to verify that the function is similar to what is expected - an integrator.



For both EAST and WEST feed forward inputs:

$$Z12_FF_source(s) := 15k$$

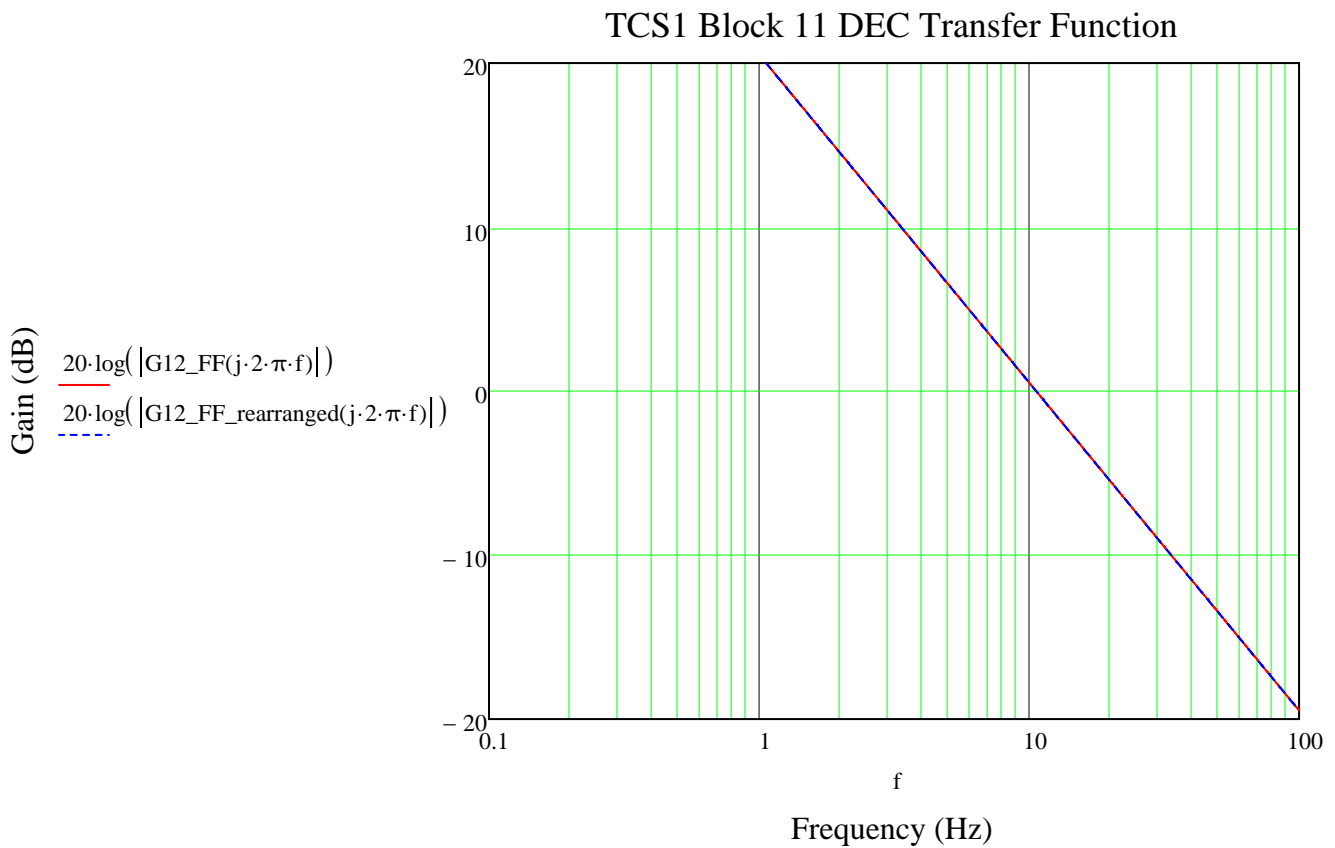
$$G12_FF(s) := \frac{-Z12_feedback(s)}{Z12_FF_source(s)}$$

$$G_{12_FF}(s) \left| \begin{array}{l} \text{float, 4} \\ \text{collect, s} \end{array} \right. \rightarrow -\frac{66.67}{s}$$

Rearrange the function into a more convenient form and verify that it is still equal to the original.

$$G_{12_FF_rearranged}(s) := -\frac{1}{0.015s}$$

Graph to verify that the function is similar to what is expected: an inverting integrator.



ALL calculations below are with R17 in parallel with the capacitor.

Generically, the transfer function for each input is:

$$G(s) = \frac{-Z_{\text{feedback}}(s)}{Z_{\text{source}}(s)}$$

Where:

$$Z12_feedback_R17(s) := \frac{\left(300 \cdot \frac{1}{s \cdot 1\mu}\right)}{300 + \frac{1}{s \cdot 1\mu}}$$

For the error input:

$$Z12_error_source(s) \rightarrow 162000$$

$$G12_error_R17(s) := \frac{-Z12_feedback_R17(s)}{Z12_error_source(s)}$$

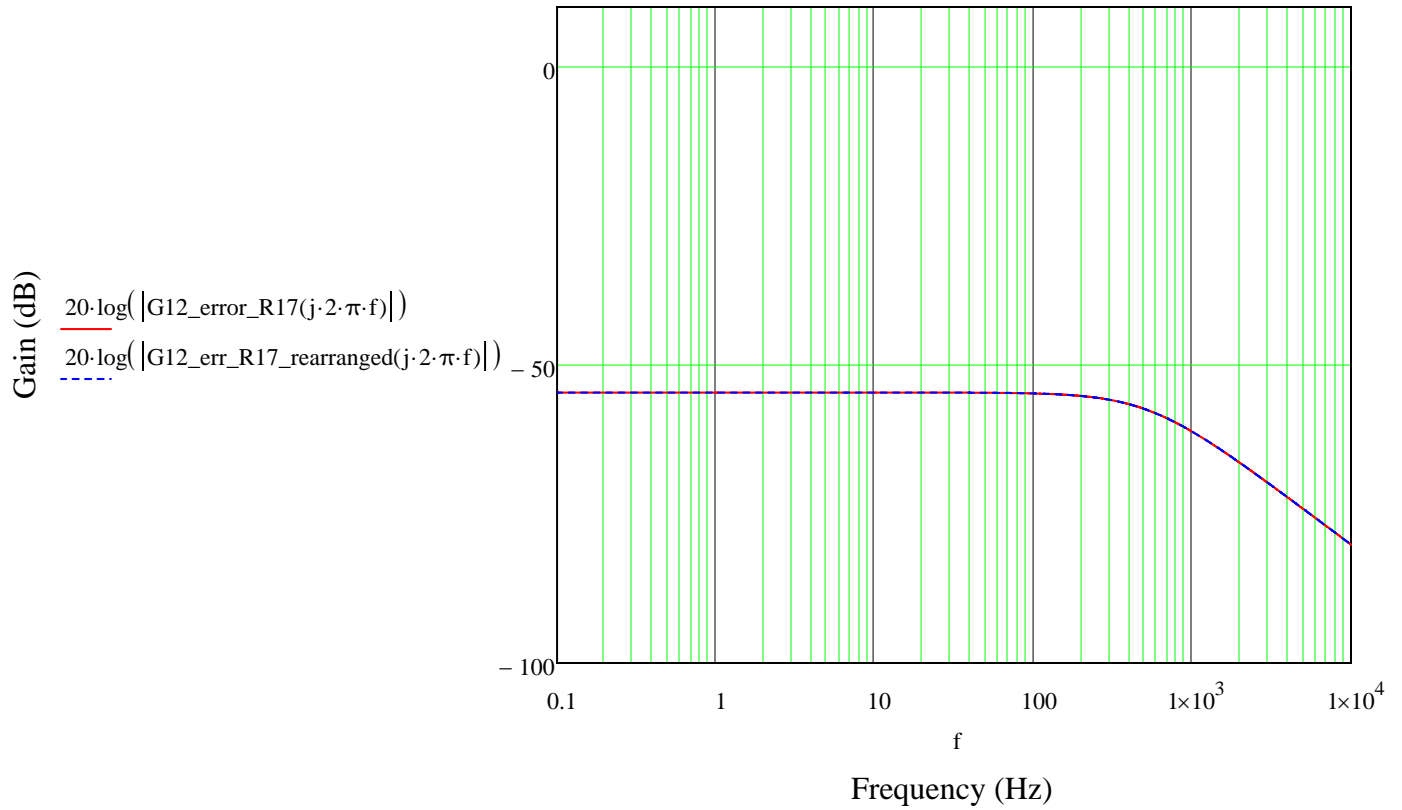
$$G12_error_R17(s) \left| \begin{array}{l} \text{float, 4} \\ \text{collect, s} \end{array} \right. \rightarrow -\frac{1.852e20}{3.0e19 \cdot s + 1.0e23}$$

Rearrange the function into a more convenient form and verify that it is still equal to the original.

$$G12_err_R17_rearranged(s) := -\frac{1.852 \cdot 10^{-3}}{300 \cdot 10^{-6} s + 1}$$

Graph to verify that the function is similar to what is expected: inverting amplifier with low gain & filter.

TCS1 Block 12 Error, R17 Transfer Function



For both EAST and WEST feed forward inputs:

$$Z12_FF_source_R17(s) := 15k$$

$$G12_FF_R17(s) := \frac{-Z12_feedback_R17(s)}{Z12_FF_source_R17(s)}$$

$$G12_FF_R17(s) \left| \begin{array}{l} \text{float, 4} \\ \text{collect, s} \end{array} \right. \rightarrow -\frac{200.0}{3.0 \cdot s + 10000.0}$$

Rearrange the function into a more convenient form and verify that it is still equal to the original.

$$G12_FF_rearranged_R17(s) := -\frac{0.02}{300 \cdot 10^{-6} s + 1}$$

Graph to verify that the function is similar to what is expected: inverting amplifier with low gain & filter.

